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Sensitivity of nuclear matter EOS parameters on neutron star properties

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Abstract : Neutron stars are composed of matter at extremely high densities. There is considerable uncertainty regarding the exact nature of neutron star matter. This uncertainty is reflected in the types of EsOS employed in theoretical study of neutron star. In this paper, a comparative study of three different types of parametric EsOS is presented. Sensitivity of parameters like compression constant at saturation (K_0) and the symmetry parameter (γ) on EOS and neutron star properties are studied and results are presented graphically.

Keywords : Neutron star, nuclear matter EOS.

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1. Introduction

Study of neutron stars is one of the important areas of modern astrophysics. Neutron stars are the smallest and densest stars known. The density in the neutron star can be up to ~7 times the equilibrium density $\rho_0 \sim 0.16$ fm^{-3} (which is equivalent to matter density $\varepsilon_0 \sim 4\times10^{14}$ gm cm^{-3}) of charged nuclear matter in nuclei. Matter at such densities has not yet been produced in the laboratory; its properties must be theoretically deduced from the available terrestrial data with guidance from observed neutron star properties. Due to this fact, study of neutron star has its importance in nuclear physics also. Neutron stars are made up of relatively cold, neutral matter that has been fully processed by nuclear combustion, so that all available energy has been exhausted at each density. Such matter in its ground state is often referred as cold catalyzed matter. The quantities of interest, in study of neutron stars, are the phase and composition of cold catalyzed neutral dense

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matter, its energy density $\varepsilon(\rho)$ and pressure $P(\rho)$, where ρ denotes the baryon number density. The baryon number is conserved in all known interactions. Therefore, it is convenient to find the composition by minimizing the total energy $\varepsilon(\rho)$ per baryon, including the rest mass contribution. This gives

$$\varepsilon(\rho) = \rho\,\varepsilon(\rho),$$

$$P(\rho) = \rho^2 \frac{\partial \varepsilon(\rho)}{\partial \rho}. \tag{1}$$

The equation of state (EOS) is found by eliminating ρ from eq. (1).

A major part of the densities inside neutron star can be well represented by nucleonic degrees of freedom only. But the EOS for neutron matter, particularly at high densities is still not very clearly known. The large uncertainties are reflected in considerable variety of significantly different parameterizations which are applied for the description of structure of neutron star [1]. As for as neutron star structure is concerned the only relevant quantity is the energy per baryon $\varepsilon(\rho)$. From $\varepsilon(\rho)$, neutron star properties such as its radius, gravitational red shift, moment of inertia *etc.* can be determined as a function of mass [1]. If we consider only the nucleonic degree of freedom for such a study of neutron star, one can adopt a simple form for binding energy per nucleon in nuclear matter consisting of a compressional term and a symmetry term. In the present work we have made a comparative study of three different forms for compressional term $\varepsilon_{\rm comp}^{\rm SN}$, $\varepsilon_{\rm comp}^{\rm Q}$, and $\varepsilon_{\rm comp}^{\rm HH}$.

The compressional term $\varepsilon_{\rm comp}^{\rm SN}$, as given by Sierk and Nix (SN) [2–4] has the following form

$$\varepsilon_{\rm comp}^{\rm SN}(\rho) = \frac{2}{9} K_0 \left(\sqrt{u} - 1\right)^2, \tag{2}$$

whereas compressional term $\varepsilon_{\rm comp}^{\rm Q}$ known as quadratic (Q) form is given by [5–7]

$$\varepsilon_{\rm comp}^{\rm Q}(\rho) = \frac{1}{18} K_0 (u-1)^2, \tag{3}$$

and $\varepsilon_{\rm comp}^{\rm HH}$ as given by Heiselberg and Hjorth-Jensen (HH) has the form [7]

$$\varepsilon_{\rm comp}^{\rm HH} = \varepsilon_0 u \frac{u-2-\delta}{1+\delta u}. \tag{4}$$

In all the three cases $u = \rho/\rho_0$ and $\rho_0 = 0.16$ fm^{-3} is saturated nuclear matter density. The compressional term at ρ_0 (*i.e.* K_0) is related to δ by $K_0 = 18\varepsilon_0/(1+\delta)$ where ε_0 = −15.8 MeV is binding energy for symmetric nuclear matter at ρ_0.

The other important term in the binding energy per nucleon in nuclear matter is the symmetry term $S(\rho)$. $S(\rho)$ is defined as the difference in energy for symmetric nuclear matter (SNM) and pure neutron matter (PNM) and can be given as

$$S(\rho) = \varepsilon\left(\rho, x_p = 0\right) - \varepsilon\left(\rho, x_p = 1/2\right), \tag{5}$$

where x_p is the proton fraction, which corresponds to ratio of protons as compared to total nucleon number (Z/A), defined as $x_p = \rho_p/\rho$. If we expand the energy per baryon in case of nucleonic degree of freedom only in the proton concentration x_p about the value of the energy for SNM (x_p = 1/2), we obtain

$$\varepsilon(\rho, x_p) = \varepsilon\left(\rho, x_p = 1/2\right) + \frac{1}{2}\frac{d^2\varepsilon(\rho)}{dx_p^2}\left(1 - 2x_p\right)^2 + \ldots, \tag{6}$$

where $d^2\varepsilon(\rho)/dx_p^2$ is to be associated with the symmetry energy $S(\rho)$ in empirical mass formula. Various studies [8–10] have revealed that the energy per baryon of a neutron-rich matter can be nicely approximated as

$$\varepsilon(\rho, x_p) = \varepsilon\left(\rho, x_p = 1/2\right) + S(\rho)\left(1 - 2x_p^2\right). \tag{7}$$

The symmetry energy term can be parameterized as [7]

$$S(\rho) = S_0 u^\gamma \left(1 - 2x_p\right)^2. \tag{8}$$

One can show that the symmetry parameter x_p can be given by

$$\rho x_p = \frac{\left(4S_0 u^\gamma (1 - 2x_p)\right)^3}{3\pi^2}. \tag{9}$$

One can have an analytical solution of eq. (9) and get x_p as function of ρ, but the soluvation as given by [7] seems to be incorrect. In the present work, we solve eq. (9) numerically which allows the inclusion of muon's contribution. For this calculation we take an empirical value of S_0 = 32 MeV [11].

Here we calculate the bulk energy using a simple form of energy density of nuclear matter consist of a compressional term, a symmetry term, and an electron energy density as depicted below.

$$\varepsilon = \rho\left[m + \varepsilon_{\text{comp}}(\rho) + S(\rho)\right] + \varepsilon_e, \tag{10}$$

where $\varepsilon_{comp}(\rho)$ can be obtained using any of the eqs. (2, 3 or 4), $S(\rho)$ can be obtained using eqs. (8, 9) and $\varepsilon_e = \mu_e^4/(12\pi^2)$ where μ_e, the chemical potential for electron is obtained using condition for charge neutrality and chemical equilibrium given as

$$\rho_p = \rho_e + \rho_\mu, \tag{11}$$

$$\mu_n = \mu_p + \mu_e, \tag{12}$$

$$\mu_e = \mu_\mu. \tag{13}$$

In all the three form of compressional term discussed before, K_0 is the parameter. We have investigated the sensitivity of gross properties of neutron star of K_0 and γ appearing in symmetry term.

Neutron star structure can be calculated by applying Einstein's general theory of relativity. The space time geometry of a non rotating, spherically symmetric star can be described by a metric function having following form in Schwarzchild coordinate as

$$ds^2 = -e^{2v(r)}dt^2 + [1-2M(r)/r]^{-1}dr^2 + r^2\left[d\Theta^2 + \sin^2\Theta d\phi^2\right]. \tag{14}$$

This equation can be reduced to well known Tolman, Oppenheimer and Volkoff (TOV) equation [7,12]

$$\frac{dP(r)}{dr} = -\frac{(\varepsilon(r)+P(r))\left(M(r)+4\pi r^3 \mathrm{P(r)}\right)}{r^2\left(1-\frac{2M(r)}{r}\right)}, \tag{15}$$

$$\frac{dM(r)}{dr} = 4\pi r^2\varepsilon(r). \tag{16}$$

Relativistic changes from Newtonian value are caused by the dragging of the local systems, the redshifts and the space-curvature.

The surface gravitational redshift and Kepler frequency is given as [12]

$$Z = \left(1-\frac{2M}{R}\right)^{-1/2} - 1, \tag{17}$$

$$\Omega_K = 22.77\left(\frac{M/M_\odot}{(R/km)^3}\right)^{1/2} \times 10^4\ \mathrm{s}^{-1}. \tag{18}$$

In the present work, we use eqs. (15–18) to calculate properties of neutron star sequence, like mass (M) radius (R), surface gravitational redshift (Z_s), and Kepler frequency (Ω_K)

2. Results and discussion

There exist, in literature, study of neutron star properties at different values of the parameters of the EsOS like K_0 and γ [1,3,4,6,7,10]. Since, there are allowed range for the numerical values for the numerical values for K_0 and γ, it is interesting to study effects of variation (sensitivity) of these parameters on EOS as well as on neutron star properties obtained using them. In this paper, we have studied the sensitivity of the parameters like compression constant at saturation (K_0) and the symmetry parameter (γ) of three different types of parametric EsOS generally employed in study of neutron stars. We define relative sensitivity of pressure on parameter $X(X = K_0, \gamma)$ as $1/P(\Delta P/\Delta X)$. In Figure 1(a) we show this sensitivity as function of K_0 for a fix value of energy density ε = 1000 MeV fm^{-3} and γ = 0.6 Similarly the relative sensitivity of pressure on γ is presented in Figure 1(b) with same energy density and K_0 = 250 MeV.

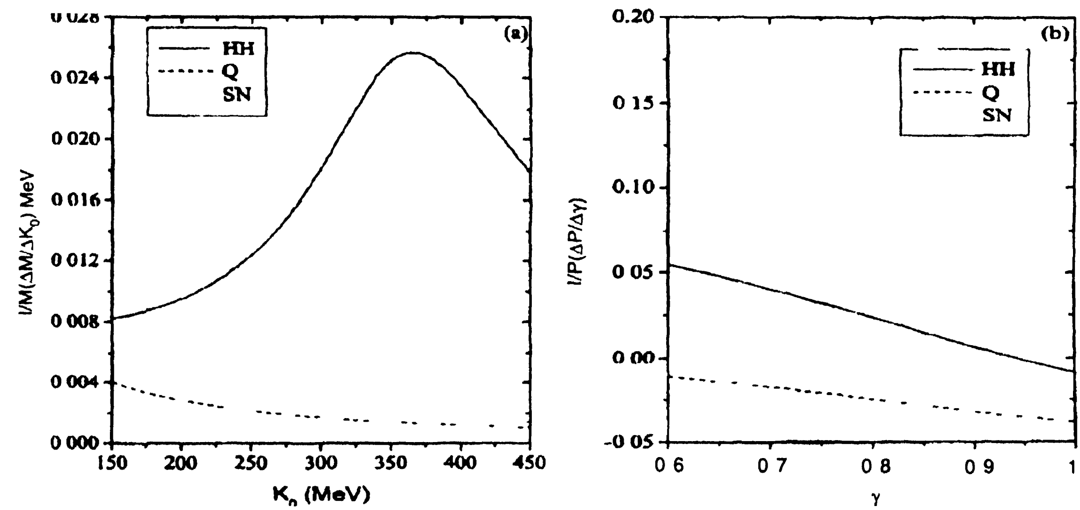

Figure 1. Relative sensitivity of pressure, (a) on K_0 at ε = 1000 MeV fm^{-3} and γ = 0.6 and (b) on γ at ε = 1000 MeV fm^{-3} and K_0 = 250 MeV

All the parametric EsOS discussed in this paper are being used to solve the TOV equations to get mass, radius and other properties of neutron star sequence. In Figure 2 (a and b) we present relative sensitivity of mass of neutron star on K_0 and γ respectively.

From Figure 1(a) we observe that the sensitivity of pressure on K_0 is always positive for all the three EsOS. Of the three EsOS discussed HH-EOS is the most sensitive to K_0 and the way this sensitivity varies, is also very different form SN and Q-EOS. While sensitivity of SN and Q-EOS show similar behavior (Figure 1(a) and 2(a)). Sensitivity of γ

on pressure found to be positive or negative depending on value of γ and ε (Figure 1(b)). SN-EOS is the most sensitive to variation in γ followed by HH and Q-EOS. Neutron star mass obtained using these EsOS shows similar variation with γ. We observe that, though pressure sensitivity on γ can be negative, mass sensitivity on γ is positive (Figure 2(b)) *i.e.* mass increases with increasing γ.

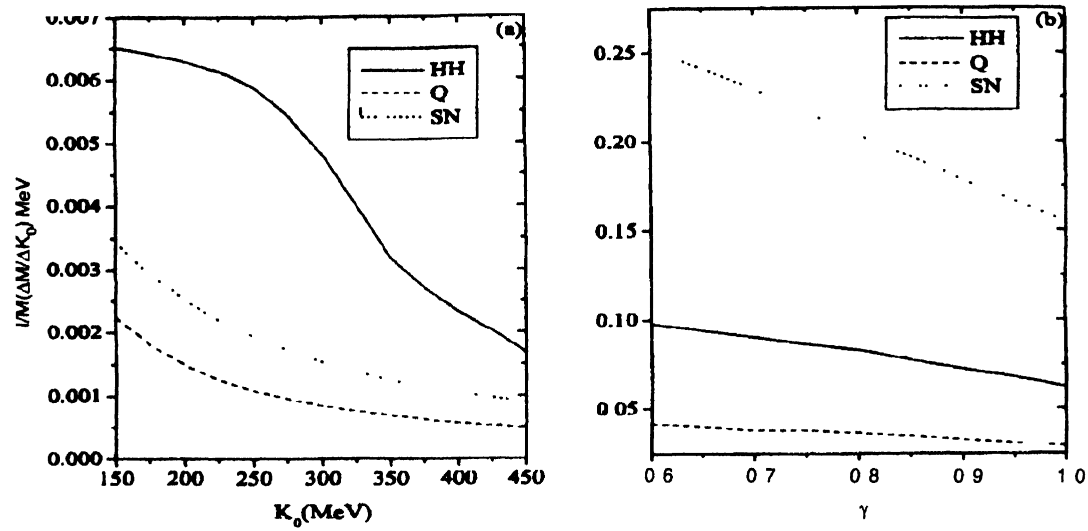

Figure 2. Relative sensitivity of masses of different neutron stars sequences at central energy density ε_c = 1000 MeV fm^{-3}, (a) for γ = 0.6 and (b) for K_0 = 250 MeV on K_0 and γ, at ε = 1000 MeV fm^{-3}.

In Figure 3, we present limiting mass properties of neutron star obtained using all the three EsOS studied in this work. In Figure 3(a) we plot maximum stable mass M_{limit}, in units of $M_\odot$, as function of K_0. In Figure 3 (b, c and d), we plot, respectively, neutron star radius (R_{limit}), surface gravitational redshift ($Z_{s(\text{limit})}$) and Kepler frequency $\left(\Omega_{K(\text{limit})}\right)$, all corresponding to maximum stable mass. From Figure 3(a) we found that, Q-EOS being most stiff among the three that gives highest value of maximum mass, but the HH-EOS is the most sensitive to variation in K_0 for mass, radius and gravitational redshift. In the case of Kepler frequency all the three parametric EsOS shows similar variation with K_0.

In Figure 4, we present limiting mass properties of neutron star as function of γ. In Figure 4 (a–d), we plot, respectively, neutron star mass (M_{limit}) radius (R_{limit}), surface gravitational redshift ($Z_{s(\text{limit})}$) and Kepler frequency $\left(\Omega_{K(\text{limit})}\right)$, all corresponding to maximum stable mass. From Figure 4(a) it can be seen that, maximum mass varies very little with γ, while Figure 4(b, c, & d) shows radius (R_{limit}), gravitational red shift ($Z_{s(\text{limit})}$) and Kepler frequency $\Omega_{K(\text{limit})}$ relatively more sensitive to variation in γ.

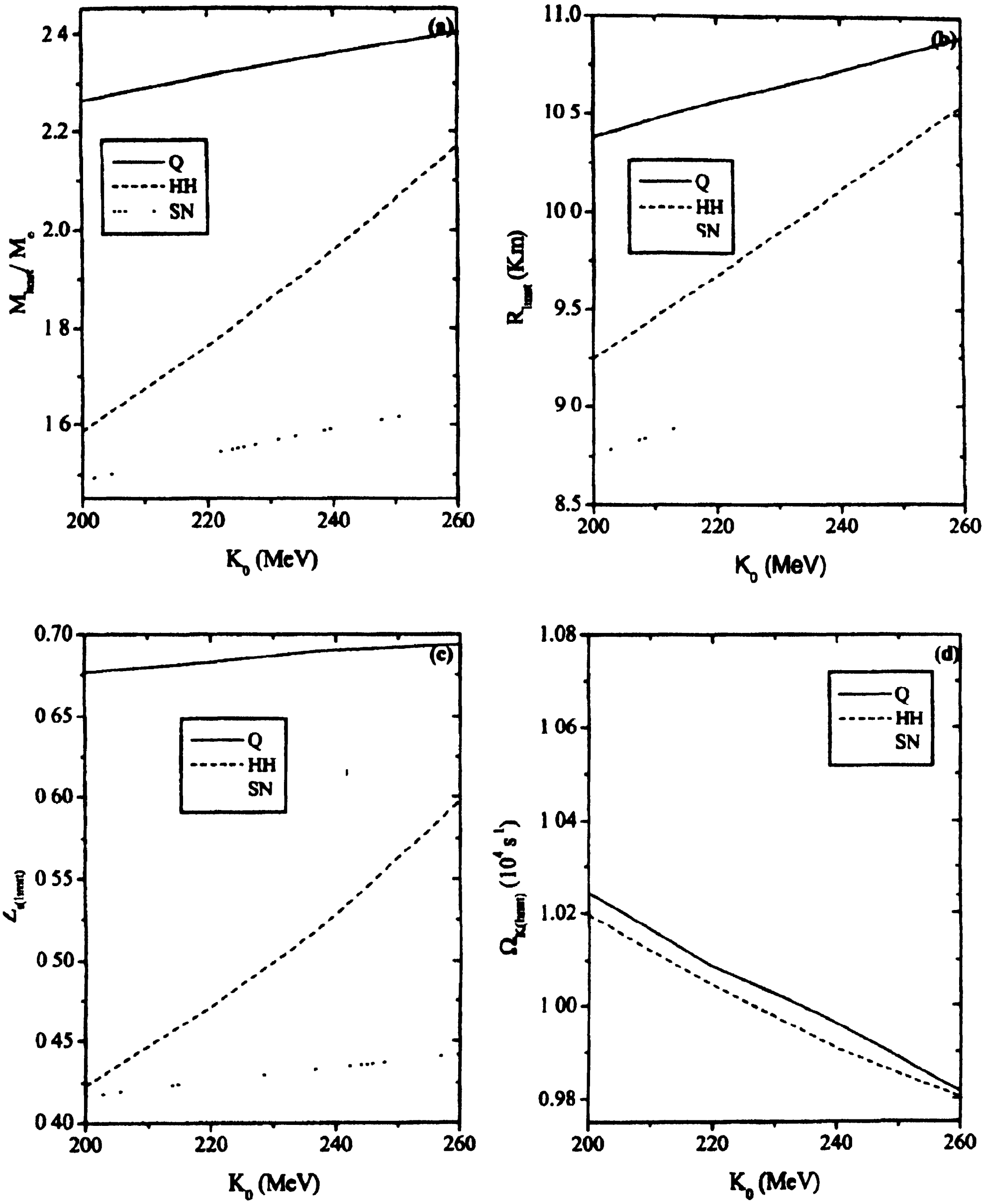

Figure 3. Variation of limiting mass properties of neutron star with K_0

3. Conclusion

From the present work we conclude that HH-EOS is the most sensitive to variation in K_0, particularly in the experimentally predicted range of K_0 (200–300 MeV). Here we find that though the expression for symmetry term is the same in all the parametric EsOS, it manifests differently. For example here we find stiff EOS is less sensitive to variation in

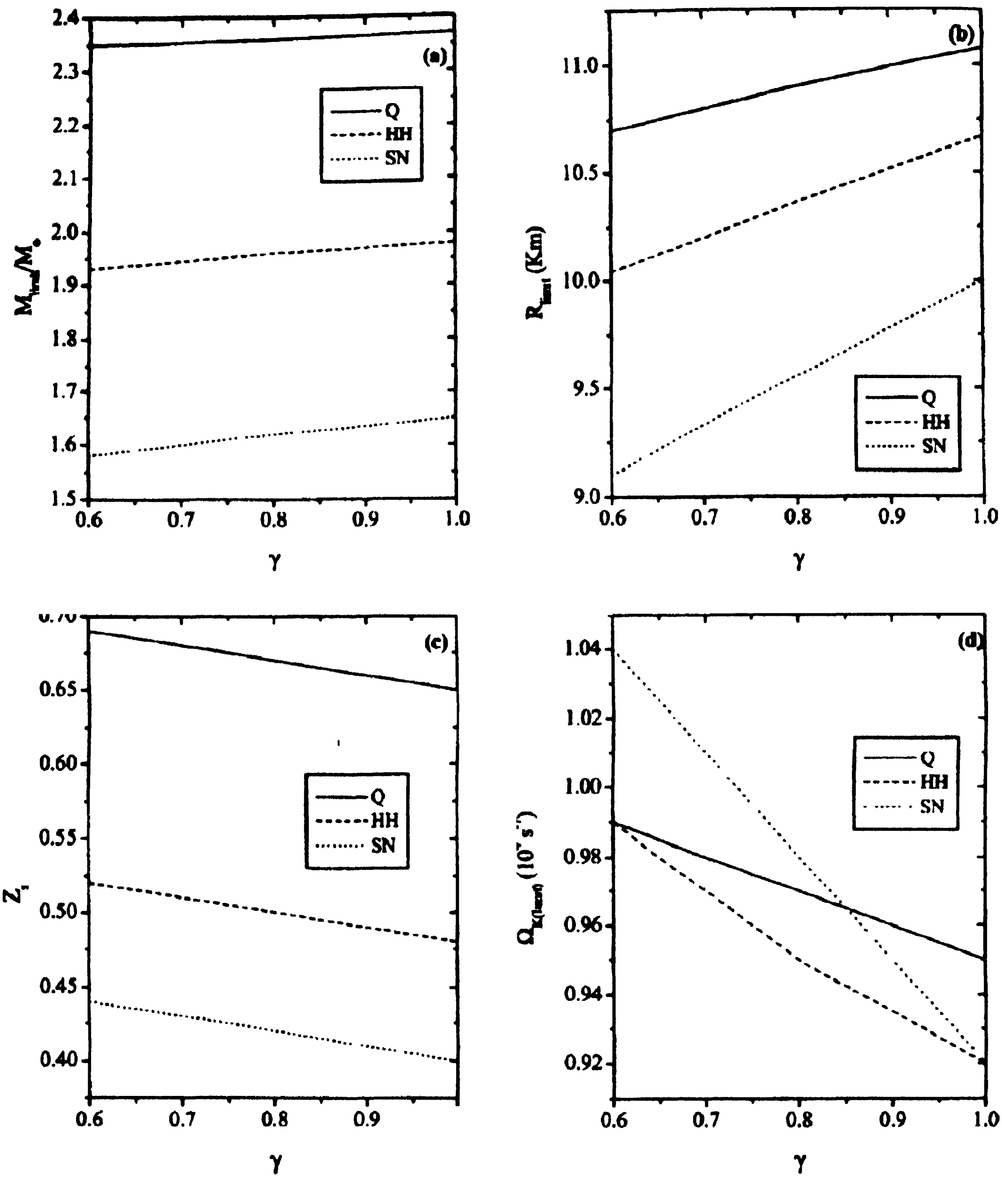

Figure 4. Variation of limiting mass properties of neutron star with γ.

γ and soft EOS is more sensitive. We also found that, variation in γ does not affect the limiting mass M_{limit} much but does have influence on R_{limit}, $Z_{s(\text{limit})}$ and $\Omega_{K(\text{limit})}$.

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